

Fundamental Theorem of Calculus & Indefinite Integrals

Section: 5.3 & 5.4

1. Solve the following definite integral using limit notation:

$$f(x) = x^2 - x + 2 \quad \text{over } [0, 3] \quad \Delta x = \frac{3}{n} \quad x_i = \frac{3}{n} i$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(\frac{3}{n} i \right)^2 - \left(\frac{3}{n} i \right) + 2 \right] \frac{3}{n}$$

$$\lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left[\frac{9}{n^2} i^2 - \frac{3}{n} i + 2 \right]$$

$$\lim_{n \rightarrow \infty} \frac{3}{n} \left[\sum_{i=1}^n \frac{9}{n^2} i^2 - \sum_{i=1}^n \frac{3}{n} i + \sum_{i=1}^n 2 \right]$$

$$\lim_{n \rightarrow \infty} \frac{3}{n} \left[\frac{9}{n^2} \left(\frac{n(n+1)(2n+1)}{6} \right) - \frac{3}{n} \left(\frac{n(n+1)}{2} \right) + 2n \right]$$

$$\lim_{n \rightarrow \infty} \frac{3}{n} \left[\frac{3(2n^2 + 3n + 1)}{2n} - \frac{3n+1}{2} + 2n \right]$$

$$\lim_{n \rightarrow \infty} \frac{3}{n} \left[\frac{6n^2}{2n} + \frac{9n}{2n} + \frac{3}{2n} - \frac{3n}{2} + \frac{3}{2} + 2n \right]$$

$$\lim_{n \rightarrow \infty} \left[\frac{18n}{2n} + \frac{9}{2} + \frac{3}{2n} - \frac{3n}{2} + \frac{3}{2} + 6n \right] = 9 - \frac{3}{2} + 6 = \boxed{\frac{21}{2} = 10.5}$$

2. Solve the above definite integral using the fundamental theorem of calculus.

$$\int_0^3 x^2 - x + 2 \, dx$$

$$\left[\frac{1}{3} x^3 - \frac{1}{2} x^2 + 2x \right]_0^3$$

$$\left[\frac{1}{3} (3)^3 - \frac{1}{2} (3)^2 + 2(3) \right] - [0]$$

$$\frac{27}{3} - \frac{9}{2} + 6 = \boxed{\frac{21}{2} = 10.5}$$

3. Solve the following integrals using the fundamental theorem of calculus:

a) $\int_1^4 2 + e^x dx$

$$[2x + e^x]_1^4$$

$$(2(4) + e^4) - (2(1) + e^1)$$

$$8 + e^4 - 2 - e$$

$$\boxed{6 + e^4 - e}$$

b) $\int_{-2}^7 x^3 - x + \frac{1}{3} dx$

$$\left[\frac{1}{4}x^4 - \frac{1}{2}x^2 + \frac{1}{3}x \right]_{-2}^7$$

$$\left[\frac{1}{4}(7)^4 - \frac{1}{2}(7)^2 + \frac{1}{3}(7) \right] - \left[\frac{1}{4}(-2)^4 - \frac{1}{2}(-2)^2 + \frac{1}{3}(-2) \right]$$

$$(600.25 - 24.5 + \frac{7}{3}) - (4 - 2 - \frac{2}{3}) = \boxed{576.75}$$

c) $\int 3x^2 + x - 5 dx$

$$\boxed{x^3 + \frac{1}{2}x^2 - 5x + C}$$

d) $\int 2 - \frac{1}{16}x + \frac{1}{5}x^2 dx$

$$\boxed{2x - \frac{1}{32}x^2 + \frac{1}{15}x^3 + C}$$

e) $\int_1^5 2 + x^{-1} - 3x^2 dx$

$$[2x + \ln|x| + 3x^{-1}]_1^5$$

$$[2(5) + \ln(5) + 3(5)^{-1}] - [2(1) + \ln(1) + 3(1)^{-1}]$$

$$10 + \ln(5) + \frac{3}{5} - 2 - 3$$

$$\boxed{5.6 + \ln(5)}$$