Calculus I SI Worksheet

Lesson 4.3: Shape of the Graph

Use the first and second derivative rules to find information about the shape of the graphs of the following functions (critical values, extrema, inflection points,etc.)

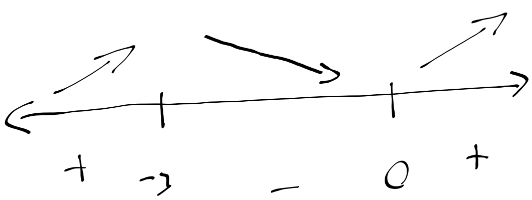
1. $f(x) = \frac{2}{3}x^3 + 3x^2 - 8$

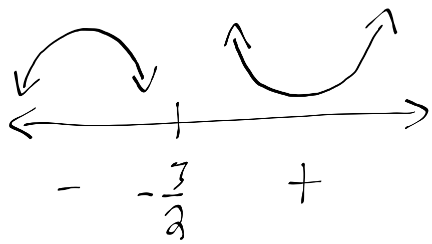
$f'(x) = 2x^2 + 6x$ $\qquad$ $f''(x) = 4x + 6$

$0 = x(2x+6)$ $\qquad$ $0 = 4x + 6$

CV @ : $x = 0, -3$

inflection points @ : $x = -\frac{3}{2}$





local min of -8 @ $x = 0$

local max of 1 @ $x = -3$

2. $f(x) = \frac{1}{4}x^4 - 2x^3 - \frac{63}{2}x^2 + 6$

$f'(x) = x^3 - 6x^2 - 63x$ $\qquad$ $f''(x) = 3x^2 - 12x - 63$

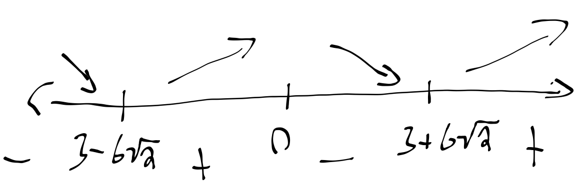
$0 = x(x^2 - 6x - 63)$ $\qquad$ $0 = 3x^2 - 12x - 63$

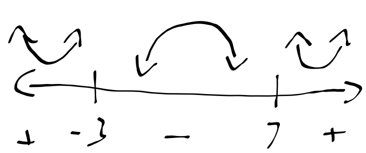
$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(-63)}}{2(1)}$ $\qquad$ $x = \dfrac{12 \pm \sqrt{(-12)^2 - 4(3)(-63)}}{2(3)}$

$x = \dfrac{6 \pm 12\sqrt{2}}{2} = 3 \pm 6\sqrt{2}$ $\qquad$ $x = \dfrac{12 \pm 30}{6} = 7, -3$

CV @ $x = 0, 3+6\sqrt{2}, 3-6\sqrt{2}$

inflection points @ $x = 7, -3$





local min of -385.37 & -2829.13 @ $x = 3 - 6\sqrt{2}$, $x = 3 + 6\sqrt{2}$ respectively

local max of 6 @ $x = 0$

3. $f(x) = e^{x^2}$

$f'(x) = 2x e^{x^2}$

$0 = 2x \boxed{e^{x^2}}$

Can never equal 0

$2x = 0$

$x = 0$

$f''(x) = 4x^2 e^{x^2} + 2 e^{x^2}$

$0 = 4x^2 \boxed{e^{x^2}} + 2 \boxed{e^{x^2}}$

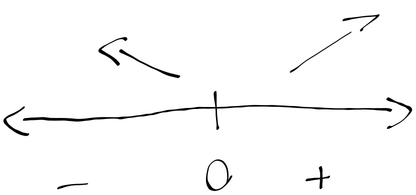
Can never equal 0, divide out

$0 = 4x^2 + 2$

$-\frac{1}{2} = x^2 \qquad x = \sqrt{-\frac{1}{2}}$ no real roots

CV @ $x = 0$

no inflection points



local min of 1 @ $x = 0$