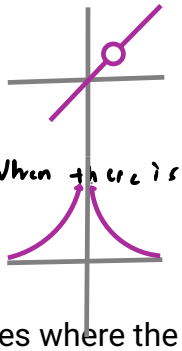
Calculus I SI Worksheet

Lesson 2.5: Continuity

1. List the three types of discontinuity, and what characterizes them.

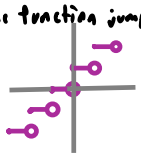
Removable Discontinuity: when you can "remove" the discontinuity by redefining a function at a certain point.



$\lim_{x \to a} f(a)$ exist

$f(a)$ DNE

Jump Discontinuity: When the function jumps from one value to another.



$\lim_{x \to a} f(a)$ exist on each side, but not overall

$f(a)$ exists

Infinite Discontinuity: When there is a vertical asymptote

$\lim_{x \to a} f(a)$ sometimes exists

$f(a)$ DNE

2. Find the values where the function is continuous given $f(x) = \frac{4x+10}{x^2-2x-15}$

$f(x)$ is continuous at

$(-\infty, -3) \cup (3, 5) \cup (5, \infty)$

$x^2 - 2x - 15$

$(x-5)(x+3) = 0$

$x = 5 \quad x = -3$

3. Given $f(x) = \frac{4x+5}{9-3x}$, is the function continuous at x=0 and x=3?

$f(0) = \frac{5}{9}$ ✓

$f(3) = \frac{17}{0}$ ✗

4. What values make f(x) = tan(2x) discontinuous?

$\tan(\theta)$ DNE @ $\frac{\pi}{2} + n\pi$

$\therefore$ $\tan(2x)$ is discontinuous

at $\boxed{x = \frac{\pi}{4} + n\frac{\pi}{2}}$

where n is any integer

5. What are the following theorems called?

If f(x) ≤ h(x) ≤ g(x), when x is near a, and the limits $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$ = L, then $\lim_{x \to a} h(x)$ = L

Squeeze Theorem

Suppose f(x) is continuous on a closed interval [a,b] and let N be any number between f(a) and f(b). Then there exists a number c in (a,b) such that f(c) = N

(IMT)

Intermediate Valve Theorem

6. Is it possible to have at least one given root on the interval [-2,4] of $f(x) = 25 - 8x^2 - x^3$

$f(x)$ is continuous

$f(-2) = 1$

$f(4) = -167$

Yes, since $f(x)$ is continuous, it must cross $y=0$ over the interval $[-2, 4]$

7. Given $4x-9 \le f(x) \le x^2 - 4x + 7$, what is $\lim_{x \to 4} f(x)$?

$\lim_{x \to 4} 4x - 9 = 7$

$\lim_{x \to 4} x^2 - 4x + 7 = 7$

$\therefore \boxed{\lim_{x \to 4} f(x) = 7}$