Calculus I SI Worksheet

Lesson 3.1 Derivative of Polynomials & Exponential Functions

Find the derivative for the following equations.

1. f(x) = $3\sqrt{x}$

$f'(x) = 3\frac{d}{dx}(x)^{1/2} = 3 \times \frac{1}{2}(x)^{-1/2}$

$$f'(x) = \frac{3}{2\sqrt{x}}$$

2. f(x) = $\frac{5}{x^2}$

$f'(x) = 5\frac{d}{dx}x^{-2} = 5 \times -2(x)^{-3}$

$$f'(x) = -\frac{10}{x^3}$$

3. f(x) = $\frac{2}{\sqrt{x^3}} + \frac{e}{x}$

$f'(x) = 2\frac{d}{dx}x^{-3/2} + e\frac{d}{dx}x^{-1} = 2 \times -\frac{3}{2}(x)^{-5/2} + e \times -1(x)^{-2}$

$$f'(x) = -\frac{3}{\sqrt{x^5}} - \frac{e}{x^2}$$

4. f(x) = $e^x - 5x^3$

$f'(x) = \frac{d}{dx}e^x - 5\frac{d}{dx}x^3 = e^x - 5 \times 3(x)^2$

$$f'(x) = e^x - 15x^2$$

5. $f(x) = \frac{x^4 - 3x^2 + 4\sqrt{x}}{\sqrt[3]{x}}$

$$f'(x) = \frac{d}{dx}\left(\frac{x^4}{x^{1/3}} - \frac{3x^2}{x^{1/3}} + \frac{4x^{1/2}}{x^{1/3}}\right) = \frac{d}{dx}x^{11/3} - 3\frac{d}{dx}x^{5/3} + 4\frac{d}{dx}x^{1/6}$$

$$f'(x) = \frac{11}{3}(x)^{8/3} - 3 \times \frac{5}{3}(x)^{2/3} + 4 \times \frac{1}{6}(x)^{-5/6}$$

$$\boxed{f'(x) = \frac{11}{3}x^{8/3} - 5x^{2/3} + \frac{2}{3}x^{-5/6}}$$

6. $f(x) = \frac{e^{2x} + e^x}{e^x}$

$$f'(x) = \frac{d}{dx}\left(\frac{e^{2x} + e^x}{e^x}\right) = \frac{d}{dx}\frac{e^x \cdot e^x}{e^x} + \frac{d}{dx}\frac{e^x}{e^x} = \frac{d}{dx}e^x + \frac{d}{dx}1$$

$$\boxed{f'(x) = e^x}$$

7. Find the normal line at x = 3 for the function $f(x) = \frac{13}{x^2} - e^x + \sqrt{x}$

$f(3) = -16.41$

$$f'(x) = 13\frac{d}{dx}x^{-2} - \frac{d}{dx}e^x + \frac{d}{dx}x^{1/2}$$

$$f'(x) = 13x^{-2}(x)^{-3} - e^x + \frac{1}{2}(x)^{-1/2}$$

$$f'(x) = \frac{-26}{x^3} - e^x + \frac{1}{2\sqrt{x}}$$

$$f'(3) = \frac{-26}{27} - e^3 + \frac{1}{2\sqrt{3}} = -20.76$$

tangent line is $\perp$ to normal

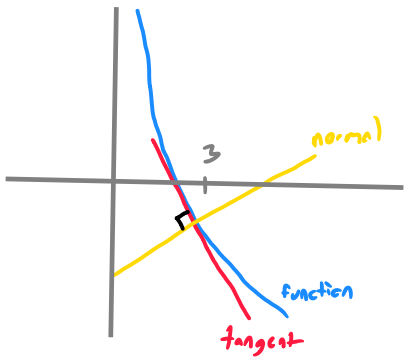
$\therefore$ m for normal is $-\frac{1}{-20.76}$

$= 0.04817$

$y - y_1 = m(x - x_1)$

$y + 16.41 = 0.04817(x - 3)$

$$\boxed{y = 0.04817x - 17.05}$$



8. Find the point on the function f(x) = $\frac{2x^{5/2} - x^2}{3\sqrt{x}}$ where the tangent line equals 0.

$$f'(x) = \frac{2}{3}\frac{d}{dx}\frac{x^{5/2}}{x^{1/2}} - \frac{1}{3}\frac{d}{dx}\frac{x^2}{x^{1/2}}$$

$$= \frac{2}{3}\frac{d}{dx}x^2 - \frac{1}{3}\frac{d}{dx}x^{3/2}$$

$$f'(x)\ \frac{2}{3}x2(x) - \frac{1}{3}x\frac{3}{2}(x)^{1/2}$$

$$f'(x) = \frac{4x}{3} - \frac{1}{2}x^{1/2}$$

$$0 = \frac{4}{3}x - \frac{1}{2}x^{1/2}$$

$$0 = x^{1/2}\left(\frac{4}{3}x^{1/2} - \frac{1}{2}\right)$$

$$x^{1/2} = 0 \qquad \frac{4}{3}x^{1/2} - \frac{1}{2} = 0$$

$$x = 0 \qquad x = \left(\frac{3}{8}\right)^2 = \frac{9}{64}$$

tangent line equals 0 at

$$\boxed{x = \frac{9}{64}}$$