Calculus I SI Worksheet

Section 3.6: Derivatives of Logarithmic Functions

1. $\frac{d}{dx} \ln(x^2)$

$$\frac{dy}{dx} = \frac{1}{x^2} \cdot 2x =$$

$$\boxed{\frac{dy}{dx} = \frac{2}{x}}$$

2. $\frac{d}{dx} \sin(3x)\log_5(x)$

$$\frac{\sin(3x) \mid \log_5(x)}{3\cos(3x) \mid \frac{1}{x \ln 5}}$$

$$\boxed{\frac{dy}{dx} = 3\cos(3x)\log_5(x) + \frac{\sin(3x)}{x \ln 5}}$$

3. $\frac{d}{dx} x^{\sin(x)}$

$$y = x^{\sin(x)}$$

$$\ln(y) = \ln(x^{\sin x})$$

$$\ln(y) = \sin(x)\ln(x)$$

Method ①

$$\frac{1}{y}\frac{dy}{dx} = \ln(x)\cos(x) + \frac{\sin(x)}{x}$$

$$\frac{dy}{dx} = \left[\ln(x)\cos(x) + \frac{\sin(x)}{x}\right] y$$

$$\boxed{\frac{dy}{dx} = \left[\ln(x)\cos(x) + \frac{\sin(x)}{x}\right] x^{\sin(x)}}$$

Method ②

$$y = e^{\sin(x)\ln(x)}$$

$$\frac{dy}{dx} = e^{\sin(x)\ln(x)}\left[\ln(x)\cos(x) + \frac{\sin(x)}{x}\right]$$

$$\left(e^{\ln(x)}\right)^{\sin(x)} = x^{\sin(x)}$$

$$\boxed{\frac{dy}{dx} = \left[\ln(x)\cos(x) + \frac{\sin(x)}{x}\right] x^{\sin(x)}}$$

4. $\frac{d}{dx}\log(\sec(x)+3)$

$$\frac{dy}{dx} = \frac{1}{(\sec(x)+3)\ln(10)}\left(\sec(x)\tan(x)\right)$$

$$\boxed{\frac{dy}{dx} = \frac{\sec(x)\tan(x)}{(\sec(x)+3)\ln(10)}}$$

5. $\frac{d}{dx}(\cot(x))^x$ $y = [\cot(x)]^x$

$\frac{1}{\cot x}\cdot(\cot x)'$

$$\ln(y) = x\ln(\cot x)$$

| $x$ | $\ln(\cot x)$ |
|---|---|
| $1$ | $\frac{-\csc^2(x)}{\cot x}$ |

$$y = e^{x\ln(\cot x)}$$

$$\frac{dy}{dx} = e^{x\ln(\cot x)}\left[\ln(\cot x) - \frac{x\csc^2(x)}{\cot(x)}\right]$$

$$\boxed{\frac{dy}{dx} = [\cot(x)]^x\left[\ln(\cot x) - \frac{x\csc^2(x)}{\cot(x)}\right]}$$

6. $\frac{d}{dx}\frac{x^2+3}{\ln(3x)}$

| $x^2+3$ | $\ln(3x)$ |
|---|---|
| $2x$ | $\frac{1}{3x}\cdot 3 = \frac{1}{x}$ |

$$\boxed{\frac{dy}{dx} = \frac{2x\ln(3x) - \frac{x^2+3}{x}}{[\ln(3x)]^2}}$$