Calculus I SI Worksheet

Exam 3 Test Prep

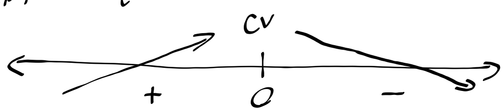
1. Find the min and max values along the given interval for the following functions:

a) f(x) = cos(x) $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$f'(x) = -\sin(x)$

$-\sin(x) = 0$ @ $x = 0 + n\pi$ on given interval, $x = 0$

$-\frac{\pi}{2}$	0
0	1
$\frac{\pi}{2}$	0



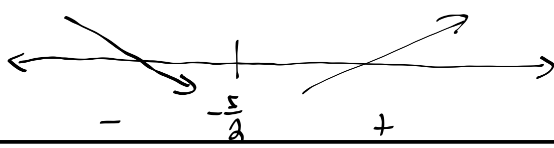
local max:	local min:	Abs. max:	abs min:
(0,1)	DNE	(0,1)	of 0 @ $x = -\frac{\pi}{2}, \frac{\pi}{2}$

b) f(x) = x² + 5x -6 [-3 , 0]

$f'(x) = 2x + 5$

$0 = 2x + 5$ $\quad x = -\frac{5}{2}$

−3	−12
$-\frac{5}{2}$	−12.25
0	−6



local max:	local min:	Abs. max:	Abs min:
DNE	$(-\frac{5}{2}, -12.25)$	(0, −6)	$(-\frac{5}{2}, -12.25)$

c) f(x) = x³ + 3x² – 72x + 5 [-5 , 5]

$f'(x) = 3x^2 + 6x - 72$

$0 = (3x + 18)(x - 4)$

$x = -6, 4$ outside of interval

−5	315
4	−171
5	−155

local max:	local min:	Abs. max	Abs. min
DNE	(4, −171)	(−5, 315)	(4, −171)

2. Find the critical numbers of the following functions:

a) f(x) = $5 - \frac{7}{5}x + \frac{1}{8}x^2$

$f'(x) = -\frac{7}{5} + \frac{1}{4}x$

$0 = \frac{1}{4}x - \frac{7}{5}$

CV @ $\boxed{x = \frac{28}{5}}$

b) f(x) = $\cos(x) - \frac{1}{2}x$

$f'(x) = -\sin(x) - \frac{1}{2}$

$0 = -\sin(x) - \frac{1}{2}$

$\sin(x) = -\frac{1}{2}$ @ $-\frac{\pi}{6} + 2n\pi$, $\frac{7\pi}{6} + 2n\pi$

CV @ $x = -\frac{\pi}{6} + 2n\pi$
$x = \frac{7\pi}{6} + 2n\pi$

3. Find all numbers that satisfy mean value theorem for: $x^3 + 2x^2 - 9x + 1$ [-4 , 4]

$f(-4) = 5$

$f(4) = 61$

$m = \frac{61 - 5}{4 - -4} = \frac{56}{8} = 7$

for closed interval $[a, b]$ of continuous function $f(x)$, their exists some number c such that

$f'(c) = \frac{f(a) - f(b)}{a - b}$

$f'(x) = 3x^2 + 4x - 9$

$7 = 3x^2 + 4x - 9$

$0 = 3x^2 + 4x - 16$

$\frac{-4 \pm \sqrt{4^2 - 4(3)(-16)}}{2(3)} = \frac{-4 \pm \sqrt{208}}{6} = \frac{-4 \pm 4\sqrt{13}}{6} = \frac{-2 \pm 2\sqrt{13}}{3}$

$\boxed{x = \frac{-2 - 2\sqrt{13}}{3}, \frac{-2 + 2\sqrt{13}}{3}}$

$\boxed{\begin{matrix}(-3.07, 18.55) \\ (1.74, -3.36)\end{matrix}}$

4. Find all numbers that satisfy Rolle's Theorem for: $\frac{x^2+x-2}{x+3}$ [-2,1]

$f(-2) = 0$

$f(1) = 0$

$m = \frac{0 - 0}{-2 - 1} = 0$

for closed interval $[a, b]$ of continuous function $f(x)$, their exists some number c such that

$f'(c) = \frac{f(a) - f(b)}{a - b} = 0$

$f'(x) = \frac{(x+3)(2x+1) - (x^2+x-2)(1)}{(x+3)^2} = \frac{(2x^2 + 7x + 3) - (x^2+x-2)}{(x+3)^2} = \frac{x^2 + 6x + 5}{(x+3)^2}$

$0 = \frac{x^2 + 6x + 5}{(x+3)^2}$

$0 = x^2 + 6x + 5 = (x+5)(x+1)$

outside interval

$x = -5, -1$

$\boxed{x = -1 \qquad (-1, -1)}$

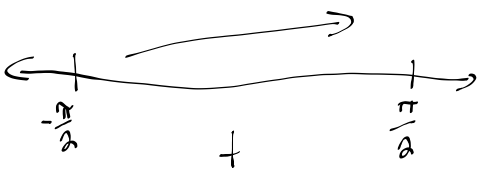
5. Use the first and second derivative rules to determine where the graph of

$$f(x) = \sin(x) \ [-\frac{\pi}{2}, \frac{\pi}{2}]$$

is increasing/decreasing and concave up/down.

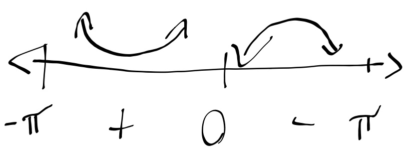
$f'(x) = \cos(x)$

$\cos(x) = 0 \quad @ \quad x = \frac{\pi}{2} + n\pi$



$f''(x) = -\sin(x)$

$-\sin(x) = 0 \quad @ \quad 0 + n\pi$



increasing:

$(-\frac{\pi}{2}, \frac{\pi}{2})$

concave up:

$[-\frac{\pi}{2}, 0)$

concave down:

$(0, \frac{\pi}{2})$

6. Use L'Hospital's Rule to evaluate the following limits:

a) $\lim_{x \to 0^+} x^x$ $= 0^0$ indeterminate

$\lim_{x \to 0^+} \quad y = x^x$

$\lim_{x \to 0^+} \left(\ln y = x \ln x = \frac{\ln x}{x^{-1}} \right)$

$\lim_{x \to 0^+} \frac{\ln x}{x^{-1}} = \frac{-\infty}{\infty}$ ind.

L'H $\lim_{x \to 0^+} \frac{\frac{1}{x}}{-x^{-2}} = \lim_{x \to 0^+} -x^2 \cdot \frac{1}{x} = \lim_{x \to 0^+} -x$

$= 0$

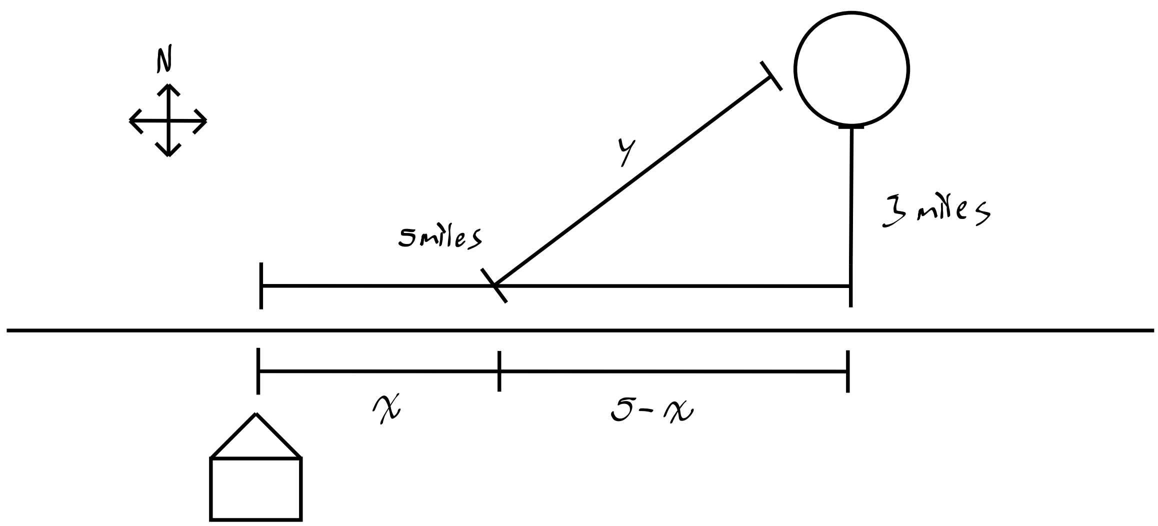
$\ln y = 0$

$y = e^0 = \boxed{1}$

b) $\lim_{x \to \infty} \frac{x+5}{3x}$ $= \frac{\infty}{\infty}$

L'H $\lim_{x \to \infty} \frac{1}{3} = \boxed{\frac{1}{3}}$

7. An island is located 3 miles from the closest point of a straight shoreline. A visitor is staying at a cabin 5 miles west of that point. The visitor plans to go from the cabin to the island. Suppose the visitor runs at a rate of 7 mph and swims at a rate of 3 mph. How far should the visitor run before swimming in order to minimize travel time?



$t_{run} = \frac{x}{7mph}$ $\quad t_{swim} = \frac{y}{3mph}$ $\quad y = \sqrt{3^2 + (5-x)^2}$

$y = \sqrt{x^2 - 10x + 34}$

$t_{total} = t_{run} + t_{swim} = \frac{x}{7} + \frac{\sqrt{x^2 - 10x + 34}}{3}$

$t' = \frac{1}{7} + \frac{1}{2} \frac{2x-10}{3\sqrt{x^2-10x+34}}$

$0 = \frac{1}{7} + \frac{1}{2} \frac{2x-10}{3\sqrt{x^2-10x+34}}$

↓

Solve for x

↓

doesn't make sense

$x = 6.423, 3.577$

$\boxed{x = 3.577}$

3.577 mile walked before swimming

8. A car rental company charges its customers p dollars per day, where $60 \leq p \leq 150$. It has found that the number of cars rented per day can be modeled by the linear function $n(p) = 750 - 5p$. How much should the company charge each customer to maximize revenue?

revenue
$\approx$
r

$r = n(p) \cdot p = 750p - 5p^2$

$r' = 750 - 10p$

$0 = 750 - 10p$

$\boxed{p = \$75}$

Max rev?

$r = n(p) \cdot p = 750p - 5p^2 = 750(75) - 5(75)^2$

$r = \$28,125$