Calculus I SI Worksheet

Exam 4 Test Prep

Find Antiderivatives

$f(x) = 6x^2 + 2x - 2$

$\boxed{F(x) = 2x^3 + x^2 - 2x + C}$

$f(x) = \dfrac{\sin(x)}{cos^2(x)} = \sec x \tan x$

$\boxed{F(x) = \sec x + C}$

$f(x) = \dfrac{-1}{\sqrt{1-x^2}}$

$\boxed{F(x) = \cos^{-1} x + C}$

$f(x) = \dfrac{x^5 - x^3 + x + 3}{x^3} = \dfrac{x^5}{x^3} - \dfrac{x^3}{x^3} + \dfrac{x}{x^3} + \dfrac{3}{x^3}$

$= x^2 - 1 + x^{-2} + 3x^{-3}$

$\boxed{F(x) = \frac{1}{3}x^3 - x - \frac{1}{x} - \frac{3}{2x^2} + C}$

Find f(x)

$f'(x) = 2x^2 + \sqrt{x}, \quad f(3) = 18$ $\quad x^{1/2}$

$f(x) = \frac{2}{3}x^3 + \frac{2}{3}x^{3/2} + C$

$18 = \frac{2}{3}(3)^3 + \frac{2}{3}(3)^{3/2} + C = 18 + 2\sqrt{3} + C$

$C = -2\sqrt{3}$

$\boxed{f(x) = \frac{2}{3}x^3 + \frac{2}{3}x^{3/2} - 2\sqrt{3}}$

$f'(x) = e^x - \sin(x) + 5, \quad f(0) = 9$

$f(x) = e^x + \cos x + 5x + C$

$9 = 1 + 1 + 0 + C$

$C = 7$

$\boxed{f(x) = e^x + \cos x + 5x + 7}$

Use the RHR to evaluate:

$f(x) = \frac{2}{3}x^3 - x^2 + 5$ [2,8] n=4

$\Delta x = \frac{8-2}{4} = 1.5$

$x_1 = 3.5$ $x_2 = 5$ $x_3 = 6.5$ $x_4 = 8$

$f(x_1) = \frac{64}{3}$ $f(x_2) = \frac{190}{3}$ $f(x_3) = \frac{875}{6}$ $f(x_4) = \frac{847}{3}$

$R_4 = 1.5\left[\frac{64}{3} + \frac{190}{3} + \frac{875}{6} + \frac{847}{3}\right]$

$\boxed{R_4 = 769.25}$

$f(x) = \cos(x) - \sin(x)$ $[0, \pi]$ n=3

$\Delta x = \frac{\pi - 0}{3} = \frac{\pi}{3}$

$x_1 = \frac{\pi}{3}$ $x_2 = \frac{2\pi}{3}$ $x_3 = \pi$

$f(x_1) = \frac{1}{2} - \frac{\sqrt{3}}{2}$ $f(x_2) = -\frac{1}{2} - \frac{\sqrt{3}}{2}$ $f(x_3) = 1 - 0$

$R_3 = \frac{\pi}{3}\left[\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}\right) + 1\right]$

$\boxed{R_3 = \frac{\pi}{3}(1 - \sqrt{3}) = -0.767}$

Consider $f(x) = 7x + 3x^2$ on the interval [0,5],

Estimate the area under the curve using n=15 and using right-hand endpoints, then evaluate using the limit definition for the function.

$$\Delta x = \frac{5-0}{15} = \frac{1}{3} \qquad x_i = \frac{1}{3} i$$

$$\lim_{n \to 15} \sum_{i=1}^{n} f\left(\tfrac{1}{3} i\right) \frac{1}{3}$$

$$\frac{1}{3} \sum_{i=1}^{15} 7\left(\tfrac{1}{3} i\right) + 3\left(\tfrac{1}{3} i\right)^2$$

$$\frac{1}{3} \sum_{i=1}^{15} \frac{7}{3} i + \frac{1}{3} i^2 = \boxed{\frac{2080}{9} = 231.1} = R_{15}$$

$$\Delta x = \frac{5-0}{n} = \frac{5}{n} \qquad x_i = \frac{5}{n} i$$

$$\lim_{n \to \infty} \sum_{i=1}^{n} f\left(\tfrac{5}{n} i\right) \frac{5}{n}$$

$$\lim_{n \to \infty} \frac{5}{n} \sum_{i=1}^{n} 7\left(\tfrac{5}{n} i\right) + 3\left(\tfrac{5}{n} i\right)^2$$

$$\lim_{n \to \infty} \frac{5}{n} \sum_{i=1}^{n} \frac{35}{n} i + \frac{75}{n^2} i^2$$

$$\lim_{n \to \infty} \frac{5}{n} \left[\frac{35}{n} \sum_{i=1}^{n} i + \frac{75}{n^2} \sum_{i=1}^{n} i^2 \right]$$

$$\lim_{n \to \infty} \frac{5}{n} \left[\frac{35}{\cancel{n}} \cdot \frac{\cancel{n}(n+1)}{2} + \frac{75}{n^{\cancel{2}}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6} \right]$$

$$\lim_{n \to \infty} \left[\frac{175}{n} \frac{(n+1)}{2} + \frac{375}{n^2} \frac{(2n^2+3n+1)}{6} \right]$$

$$\lim_{n \to \infty} \left[\frac{175\cancel{n}}{2\cancel{n}} + \underbrace{\frac{175}{2n}}_{0} + \frac{750\cancel{n^2}}{6\cancel{n^2}} + \underbrace{\frac{1125n}{6n^3}}_{0} + \underbrace{\frac{375}{6n^3}}_{0} \right]$$

$$\frac{175}{2} + \frac{750}{6} = \boxed{212.5}$$

check

$$\int_0^5 7x + 3x^2 \, dx$$

$$\frac{7}{2}x^2 + x^3 \Big|_0^5$$

$$\frac{7}{2}(5)^2 + (5)^3 - 0 - 0$$

$$= 212.5$$

Consider $f(x) = x^2 - x + 3$ on the interval [2,5] using the limit definition

$\Delta x = \frac{5-2}{n} = \frac{3}{n} \qquad \Delta x_i = \frac{3}{n}i + 2$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[\left(\frac{3}{n}i+2\right)^2 - \left(\frac{3}{n}i+2\right) + 3\right]$$

expand

$$\left(\frac{3}{n}i+2\right)\left(\frac{3}{n}i+2\right)$$
$$\frac{9}{n^2}i^2 + \frac{6}{n}i + \frac{6}{n}i + 4$$

$$\lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n} \frac{9}{n^2}i^2 + \frac{12}{n}i + 4 - \frac{3}{n}i - 2 + 3\Big]$$

combine like terms

$$\lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n} \frac{9}{n^2}i^2 + \frac{9}{n}i + 5$$

$$\lim_{n\to\infty} \frac{3}{n}\left[\frac{9}{n^2}\sum_{i=1}^{n} i^2 + \frac{9}{n}\sum_{i=1}^{n} i + \sum_{i=1}^{n} 5\right]$$

$$\lim_{n\to\infty} \frac{3}{n}\left[\frac{9}{n^{\cancel{2}}}\,\frac{\cancel{n}(n+1)(2n+1)}{6} + \frac{9}{\cancel{n}}\,\frac{\cancel{n}(n+1)}{2} + 5n\right]$$

$$\lim_{n\to\infty} \left[\frac{27}{n^2}\,\frac{(2n^2+3n+1)}{6} + \frac{27}{n}\,\frac{(n+1)}{2} + \frac{15n}{n}\right]$$

$$\lim_{n\to\infty} \left[\frac{54\cancel{n^2}}{6\cancel{n^2}} + \underbrace{\frac{81\cancel{n}}{6n^{\cancel{2}}}}_{0} + \underbrace{\frac{27}{6n^2}}_{0} + \frac{27\cancel{n}}{2\cancel{n}} + \underbrace{\frac{27}{2n}}_{0} + \frac{15\cancel{n}}{\cancel{n}}\right]$$

$$\frac{54}{6} + \frac{27}{2} + 15 = \boxed{37.5}$$

$$\int_2^5 x^2 - x + 3 \, dx$$

$$\frac{1}{3}x^3 - \frac{1}{2}x^2 + 3x\Big|_2^5$$

$$\frac{1}{3}(5)^3 - \frac{1}{2}(5)^2 + 3(5) - \frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 - 3(2)$$

$$\frac{125}{3} - \frac{25}{2} + 15 - \frac{8}{3} + \frac{4}{2} - 6$$

$$39 - \frac{21}{2} + 9 = \underline{37.5}$$

Find f'(x) for the following function using the fundamental theorem of calculus pt. 2

$$f(x) = \int_0^x \sqrt{8 - 2t^2}\,dt$$

$$\boxed{f'(x) = \sqrt{8 - 2x^2}}$$

$$f(x) = \int_{3x^2}^{0} \theta^2 - e^{\theta}\,d\theta$$

$$f'(x) = \left[(3x^2)^2 - e^{3x^2}\right](3x^2)' \quad 6x$$

$$f'(x) = 6x(9x^4) - 6xe^{3x^2}$$

$$\boxed{f'(x) = 54x^5 - 6xe^{3x^2}}$$

Evaluate the definite integral, $\int_3^5 e^x - \cos(x)\,dx$ using the fundamental theorem of calculus pt. 1

$$e^x + \sin(x) \Big|_3^5$$

radians

$$\boxed{e^3 + \sin 3 - e^5 - \sin 5 \approx -127.23}$$

Evaluate the following indefinite integrals.

$\int 3x^2 - 14x + \sqrt{5}dx$

$$\boxed{x^3 - 7x^2 + \sqrt{5}\,x + C}$$

$\int \sqrt{x} - x^{\frac{3}{2}}$

$$\boxed{\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C}$$

Evaluate the following integrals

$\int_1^3 \frac{3+\cos(x)}{\sqrt{3x+\sin(x)}}dx$

$u = 3x + \sin x$

$du = (3 + \cos x)\,dx$

$\int_{\sim}^{\sim} \frac{1}{\sqrt{u}}\,du$

$2\sqrt{u}\,\Big|_{\sim}^{\sim}$

$2\sqrt{3x+\sin x}\,\Big|_1^3$

$$\boxed{2\sqrt{9+\sin 3} - 2\sqrt{3+\sin 1} \approx 2.13}$$

$\int_2^5 3xe^{x^2}dx$

$u = x^2$

$du = 2x\,dx$

x	u
2	4
5	25

$\int_2^5 \frac{3}{2}e^{x^2}(2x)\,dx$

$\int_4^{25} \frac{3}{2}e^u\,du$

$\frac{3}{2}e^u\Big|_4^{25}$

$$\boxed{\frac{3}{2}e^{25} - \frac{3}{2}e^4 \approx 1.08 \times 10^{11}}$$