U-Substitution Rule

Section: 5.5

Evaluate the indefinite integrals:

$$\int 4x\sqrt{3-x^2}\,dx$$

$u = 3 - x^2$

$du = 2x\,dx$

$\int -\sqrt{3-x^2}\,(2)(2x)\,dx$

$2\int -\sqrt{u}\,du$

$2\int -u^{1/2}\,du$

$(2)\frac{2}{3}u^{3/2}+C$

$\frac{4}{3}(u)^{3/2}+C$

$\boxed{\frac{4}{3}(3-x^2)^{3/2}+C}$

$$\int \frac{-\cos(2x)}{\sin^2(2x)}\,dx$$

$u = \sin(2x)$

$du = 2\cos(2x)\,dx$

$\frac{du}{2\cos(2x)} = dx$

$\int \frac{-\cancel{\cos(2x)}}{u^2}\cdot\frac{du}{2\cancel{\cos(2x)}}$

$-\frac{1}{2}\int u^{-2}\,du$

$-\frac{1}{2}\left(-u^{-1}\right)+C$

$\frac{1}{2u}+C =$

$\boxed{\frac{1}{2\sin(2x)}+C}$

$$\int \frac{6x^2}{\sqrt{5+2x^3}}\,dx$$

$u = 5+2x^3$

$du = 6x^2\,dx$

$\int \frac{1}{\sqrt{u}}\,du$

$\int (u)^{-1/2}\,du$

$2(u)^{1/2}+C$

$\boxed{2\sqrt{5+2x^3}+C}$

Evaluate the following definite integrals:

$$\int_5^6 3x^5\sqrt{1+x^2}\,dx$$

$u = 1 + x^2$ $\quad u-1 = x^2$

$du = 2x\,dx$ $\quad (u-1)^2 = x^4$

$\frac{du}{2x} = dx$

u	x
26	5
37	6

$\int_{26}^{37} \sqrt{u}\,(x^4)(3x)\,\frac{du}{2x}$

$\frac{3}{2}\int_{26}^{37} (u)^{1/2}(u-1)^2\,du$

$\frac{3}{2}\int_{26}^{37} u^{1/2}(u^2-2u+1)\,du$

$\frac{3}{2}\int_{26}^{37} u^{5/2}-2u^{3/2}+u^{1/2}\,du$

$\frac{3}{2}\left(\frac{2}{7}u^{7/2}-\frac{4}{5}u^{5/2}+\frac{2}{3}u^{3/2}\right)\Big|_{26}^{37}$

$\approx (122\,279.52 - 34\,404.98)$

$\boxed{\approx 87\,874.54}$

$$\int_{-1}^{2}\sqrt{3+4x}\,dx$$

$u = 3+4x$

$du = 4\,dx$

$\frac{du}{4} = dx$

u	x
7	1
11	2

$\int_7^{11} \sqrt{u}\,d\frac{u}{4}$

$\frac{1}{4}\int_7^{11} (u)^{1/2}\,du$

$\frac{1}{4}\left(\frac{2}{3}u^{3/2}\right)\Big|_7^{11}$

$\frac{1}{6}(11)^{3/2} - \frac{1}{6}(7)^{3/2}$

$\boxed{\frac{\sqrt{11^3}}{6} - \frac{\sqrt{7^3}}{6} \approx 2.994}$

$$\int_0^{\frac{\pi}{3}} \cos(2x)e^{\sin(2x)}\,dx$$

$u = \sin(2x)$

$du = 2\cos(2x)\,dx$

$\frac{du}{2\cos(2x)} = dx$

u	x
0	0
$\frac{\sqrt{3}}{2}$	$\pi/3$

$\int_0^{\sqrt{3}/2} e^u\,\cancel{\cos(2x)}\,\frac{du}{2\cancel{\cos(2x)}}$

$\frac{1}{2}\int_0^{\sqrt{3}/2} e^u\,du$

$\frac{1}{2}e^u\Big|_0^{\sqrt{3}/2}$

$\boxed{\frac{e^{\sqrt{3}/2}}{2} - \frac{1}{2} \approx 0.689}$