Calculus I SI Worksheet

Lesson 4.4: L'Hospital's Rule

1. $\lim_{x \to \frac{\pi}{2}} \frac{1+\sin(3x)}{1-\sin(x)}$ $= \frac{1+\sin\left(\frac{3\pi}{2}\right)}{1-\sin\left(\frac{\pi}{2}\right)} = \frac{1-1}{1-1} = \frac{0}{0}$ indet.

L'H $\lim_{x \to \frac{\pi}{2}} \frac{3\cos(3x)}{-\cos(x)} = \frac{3\cos\left(\frac{3\pi}{2}\right)}{-\cos\left(\frac{\pi}{2}\right)} = \frac{0}{0}$ indet.

L'H $\lim_{x \to \frac{\pi}{2}} \frac{-9\sin(3x)}{\sin(x)} = \frac{-9\sin\left(\frac{3\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = \frac{-9(-1)}{1} = \boxed{9}$

2. $\lim_{x \to 0} [\sin(x)]^x$ $= 0^0$ indet.

$\lim_{x \to 0} \left[y = \sin(x)^x \right]$

$\lim_{x \to 0} \left[\ln y = \ln \sin(x)^x \right]$

$= \lim_{x \to 0} x \ln \sin x = 0 \cdot -\infty$ indet.

$= \lim_{x \to 0} \frac{\ln \sin x}{x^{-1}} = \frac{-\infty}{\infty}$ indet.

L'H $= \lim_{x \to 0} \frac{\frac{1}{\sin x} \cdot \cos x}{-1x^{-2}} = \lim_{x \to 0} \frac{-x^2 \cos x}{\sin x} = \frac{0 \cdot 1}{0} = \frac{0}{0}$ indet.

L'H $= \lim_{x \to 0} \frac{-2x\cos x + x^2 \sin x}{\cos x} = \frac{0 \cdot 1 + 0 \cdot 0}{1} = \frac{0}{1} = \underline{0}$

* $\ln y = 0$

$y = e^0 = \boxed{1}$

3. $\lim_{x\to 0^+} \frac{1}{x} - \csc(x) = \infty - \infty$ indet.

$$\lim_{x\to 0^+} \frac{1}{x} - \frac{1}{\sin x}$$

$$\lim_{x\to 0^+} \frac{\sin x - x}{x \sin x} = \frac{0-0}{0\cdot 0} = \frac{0}{0} \text{ indet.}$$

L'H $$\lim_{x\to 0^+} \frac{\cos x - 1}{x\cos x + \sin x} = \frac{1-1}{0\cdot 1 + 0} = \frac{0}{0} \text{ indet}$$

L'H $$\lim_{x\to 0^+} \frac{-\sin x}{\cos x - x\sin x + \cos x} = \frac{0}{1 - 0\cdot 0 + 1} = \frac{0}{2} = \boxed{0}$$