Calculus I SI Worksheet

Exam 1 Test Prep

1. The point P(8,5) lies on the graph of $f(x) = 7 - x^{1/3}$. If Q is another point on the function, use your calculator to find the slope of the secant line, PQ, for the following x-coordinate for Q.

| P | Q | m(PQ) |
|---|---|---|
| ( 8 , 5 ) | ( 7.9 , 5.008368 ) | -0.0837 |
| ( 8 , 5 ) | ( 7.99 , 5.000834 ) | -0.0834 |
| ( 8 , 5 ) | ( 7.999 , 5.000083 ) | -0.0833 |

Use the result above to estimate the slope of the tangent line of f(x) at the point P(8,5).

The slope of the tangent line is

$$\boxed{-0.0833 \text{ or } -1/12}$$

Finally, write the equation of the tangent line at point P(8,5).

$$y - y_0 = m(x - x_0)$$

$$y - 5 = -\frac{1}{12}(x - 8)$$

$$\boxed{y = -\frac{1}{12}x + \frac{17}{3}}$$

2. Use a table of values to evaluate the following limit:

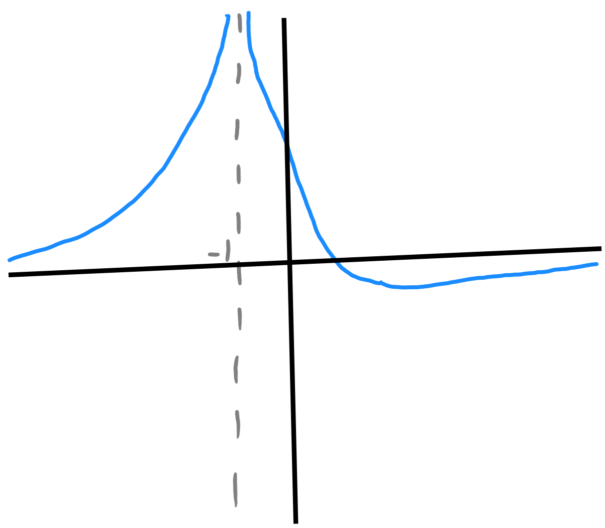
$$\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{x-\frac{\pi}{2}}$$

| Left | | Right | |
|---|---|---|---|
| in | Out | in | Out |
| $\frac{\pi}{4}$ | -0.900 | $\frac{3\pi}{4}$ | -0.900 |
| $\frac{2\pi}{5}$ | -0.984 | $\frac{3\pi}{5}$ | -0.984 |
| $\frac{3\pi}{7}$ | -0.992 | $\frac{4\pi}{7}$ | -0.992 |

$$\lim_{x\to\frac{\pi}{2}} \frac{\cos(x)}{x-\frac{\pi}{2}} = -1$$

3. Use Graphical techniques to evaluate the following limit (if it exists).

$$\lim_{x\to-1} \frac{1-x}{(x+1)^2}$$




$$\lim_{x\to-1^-} \frac{1-x}{(x+1)^2} = \infty$$

$$\lim_{x\to-1^+} \frac{1-x}{(x+1)^2} = \infty$$

$$\boxed{\lim_{x\to-1} \frac{1-x}{(x+1)^2} = \infty}$$

4. Use limit laws to evaluate the following limit (if it exists):

$$\lim_{x \to 7} \frac{x^3 - 343}{x - 7}$$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$\lim_{x \to 7} \frac{x^3 - 7^3}{x - 7}$$

$$\lim_{x \to 7} \frac{\cancel{(x-7)}(x^2 + 7x + 49)}{\cancel{x-7}}$$

$$\lim_{x \to 7} x^2 + 7x + 49 = \boxed{147}$$

5. Use limit laws to evaluate the following limit (if it exists):

$$\lim_{x \to 9} \frac{3 - \sqrt{x}}{9 - x} \quad \frac{(3 + \sqrt{x})}{(3 + \sqrt{x})}$$

$$\lim_{x \to 9} \frac{\cancel{9-x}}{\cancel{(9-x)}(3 + \sqrt{x})}$$

$$\lim_{x \to 9} \frac{1}{3 + \sqrt{x}} = \boxed{\frac{1}{6}}$$

6. Use limit laws to evaluate the following limit (if it exists):

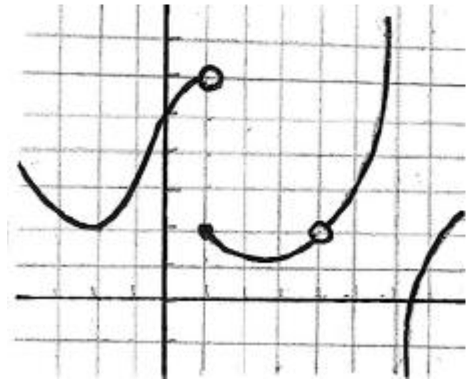
$$\lim_{h \to 0} \frac{(2 + h)^2 - 6(2 + h) + 8}{h}$$

$$\lim_{h \to 0} \frac{\cancel{4} + 4h + h^2 - \cancel{12} - 6h + \cancel{8}}{h}$$

$$\lim_{h \to 0} \frac{4\cancel{h} + h^{\cancel{2}} - 6\cancel{h}}{\cancel{h}}$$

$$\lim_{h \to 0} h - 2 = \boxed{-2}$$

7. For the function f(x) whose graph is given, state the value of each quantity.



a) $\lim_{x \to -2} f(x)$

2

e) $\lim_{x \to 4} f(x)$

2

b) $\lim_{x \to 1^-} f(x)$

6

f) $\lim_{x \to 6^-} f(x)$

$+\infty$

c) $\lim_{x \to 1^+} f(x)$

2

g) $\lim_{x \to 6} f(x)$

DNE

d) $\lim_{x \to 1} f(x)$

DNE

h) $f(1)$

2

8. A machinist is required to manufacture a circular metal disk with an area of 500cm$^2$.

a) what is the ideal radius required to manufacture a dish with these specifications?

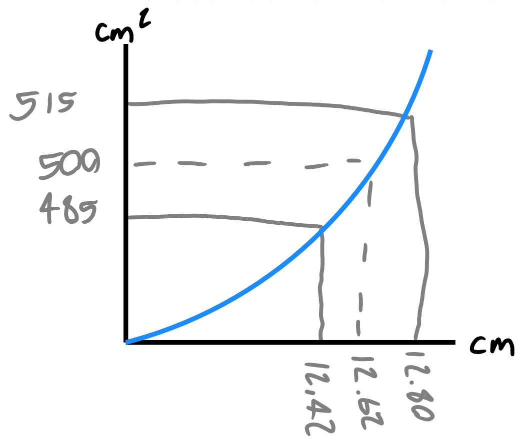
$$A = \pi r^2$$

$$500cm^2 = \pi r^2$$

$$r = \sqrt{\frac{500cm}{\pi}}$$

$$\boxed{radius = 12.62\ cm}$$

b) If the machinist is allowed an error tolerance of ±15cm$^2$ in the are of the disk, how close to the ideal radius must the machinist control the radius?




$\delta_R = 0.18$

$\delta_L = 0.20$

$\delta = 0.18$

c) Use the precise definition of a limit to describe the above problem.

if $0 < |r - 12.62| < 0.18$

then $|\pi r^2 - 500| < 15$

9. Find the values where f(x) is continuous given:

$$f(x) = \frac{x-5}{x^2+x-6}$$

discontinuous when denominator $=0$

$x^2+x-6=0$

$(x-2)(x+3)=0 \qquad x=2,-3$

$$\boxed{(-\infty,-3)\cup(-3,2)\cup(2,\infty)}$$

10. $\lim_{t\to 1} \frac{4t-6}{2-t^2}$

$$\frac{4(1)-6}{2-(1)^2} = \frac{-2}{1} = \boxed{-2}$$

11. $\lim_{x\to 0} \frac{x}{4-\sqrt{x+16}}$

$$\lim_{x\to 0} \frac{x}{4-\sqrt{x+16}} \cdot \frac{(4+\sqrt{x+16})}{(4+\sqrt{x+16})} = \lim_{x\to} \frac{x(4+\sqrt{x+16})}{\cancel{16}-(x+\cancel{16})}$$

$$\lim_{x\to 0} \frac{\cancel{x}(4+\sqrt{x+16})}{-\cancel{x}} = \lim_{x\to 0} -(4+\sqrt{x+16})$$

$$= -4-\sqrt{16} = \boxed{-8}$$

12. Use the limit definition to find the derivative of:

$$f(x) = \frac{x-4}{2x+1}$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to a} \frac{\frac{x-4}{2x+1} - \frac{a-4}{2a+1}}{x-a}$$

$$\lim_{x \to a} \frac{(x-4)(2a+1) - (a-4)(2x+1)}{(x-a)(2x+1)(2a+1)}$$

$$\lim_{x \to a} \frac{\cancel{2ax} + x - 8a - \cancel{4} - \cancel{2ax} - a + 8x + \cancel{4}}{(x-a)(2x+1)(2a+1)}$$

$$\lim_{x \to a} \frac{9\cancel{(x-a)}}{\cancel{(x-a)}(2x+1)(2a+1)}$$

$$\lim_{x \to a} \frac{9}{(2x+1)(2a+1)} = \boxed{\frac{9}{(2a+1)^2}}$$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\lim_{h \to 0} \frac{\frac{a+h-4}{2(a+h)+1} - \frac{a-4}{2a+1}}{h}$$

$$\lim_{h \to 0} \frac{(a+h-4)(2a+1) - (a-4)(2a+2h+1)}{h[2(a+h)+1](2a+1)}$$

$$\lim_{h \to 0} \frac{\cancel{2a^2} + \cancel{a} + \cancel{2ah} + h - \cancel{8a} - \cancel{4} - \cancel{2a^2} - \cancel{2ah} - \cancel{a} + \cancel{8a} + 8h + \cancel{4}}{h[2(a+h)+1](2a+1)}$$

$$\lim_{h \to 0} \frac{9\cancel{h}}{\cancel{h}[2(a+h)+1](2a+1)}$$

$$\lim_{h \to 0} \frac{9}{[2(a+h)+1](2a+1)} = \boxed{\frac{9}{(2a+1)^2}}$$