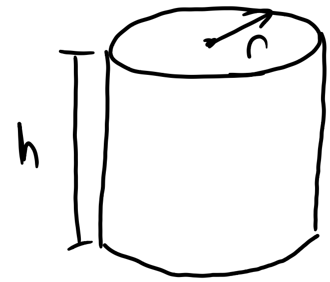
Calculus I SI Worksheet

Lesson 4.7: Optimization

A client hired your company to manufacture cylindrical cans out of aluminum. The total surface area of the can needs to be 30 square inches. What is the largest possible volume of such a can? What radius should you calibrate your machines for?

Known:

$SA = 30 \text{ in}^2$



$V = \pi r^2 h$ $\qquad SA = 2\pi r^2 + 2\pi r h$

$30 = 2\pi r^2 + 2\pi r h$

$h = \dfrac{30 - 2\pi r^2}{2\pi r}$

$V = \cancel{\pi} r^{\cancel{2}} \left(\dfrac{\overset{15}{\cancel{30}} - 2\pi r^2}{\cancel{2\pi r}} \right)$

$V = r(15 - \pi r^2)$

$V = 15r - \pi r^3$

$V' = 15 - 3\pi r^2$

Find Critical Values

$0 = 15 - 3\pi r^2$

$\pm\sqrt{\dfrac{5}{\pi}} = \boxed{r \approx 1.26 \text{ in}}$

radius of can

* can't have negative radius

Value of r to find h

$h = \dfrac{30 - 2\pi r^2}{2\pi r}$

$= \dfrac{30 - 2\pi\left(\sqrt{\frac{5}{\pi}}\right)^2}{2\pi\left(\sqrt{\frac{5}{\pi}}\right)} \approx 2.52 \text{ in}$

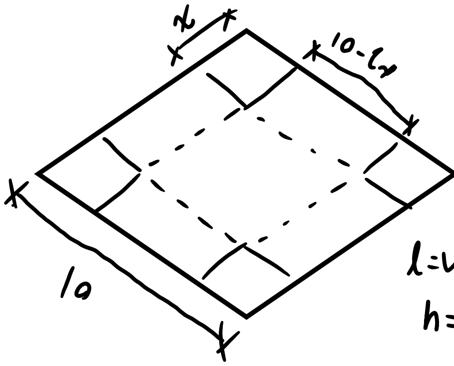
Maximized volume:

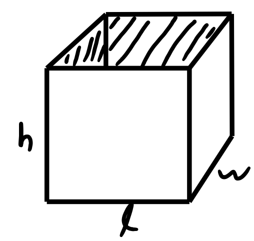
$V = \pi r^2 h = \pi(1.26)^2(2.52)$

$\boxed{V_{max} = 12.62 \text{ in}^3}$

rectangular prism

A storage container manufacturer is designing a new storage container. The containers will be a ~~cube~~ with no top face. Each container will be manufactured by cutting the corners off of a 10 meter by 10 meter sheet of metal. What is the largest volume these containers can hold?





$l = w = 10-2x$

$h = x$

$V = l \cdot w \cdot h$

$V = (10-2x)^2(x)$

$V = (100-40x+4x^2)x$

$V = 4x^3-40x^2+100x$

$V' = 12x^2-80x+100$

Critical values

$0 = 12x^2-80x+100$

$0 = 4(3x^2-20x+25)$

$0 = 4(3x-5)(x-5)$

$x = \frac{5}{3}, 5$

Which x value is better?

$V = (10-2x)^2 x$

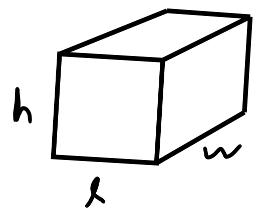
$V(\frac{5}{3}) = (10-2(\frac{5}{3}))^2 \frac{5}{3}$ $\qquad$ $V(5) = (10-2(5))^2(5)$

$\boxed{V(\frac{5}{3}) = 74.07}$ $\qquad$ $V(5) = 0$

A client orders a box with a lid from your company. The material for the box cost \$8 per square foot. The client requires the length of the box to be twice its width, and it must have a volume of 65 cubic feet. Find the most cost-effective box and report its price.

$l = 2w$

$V = 65\ ft^3$



$V = l \cdot w \cdot h$

$65 = 2w^2 h$

$h = \frac{65}{2w^2}$

$SA = 2lw + (2l + 2w)h$

$SA = 4w^2 + (6\cancel{w})\left(\frac{65}{2w^{\cancel{2}}}\right)$

$SA = 4w^2 + 195\ w^{-1}$

$SA' = 8w - 195w^{-2}$

CV's

$0 = 8w - 195\ w^{-2}$

$0 = w^{-2}(8w^3 - 195)$

$w = \sqrt[3]{\frac{195}{8}} \approx 2.90)$

Solve for price

$SA = 4w^2 + 195w^{-1}$

$= 4(2.90)^2 + 195(2.90)^{-1}$

$SA \approx 100.9\ ft^2.$

$100.9\ ft^2 \cdot \frac{\$8}{1ft} \approx \boxed{\$807.05}$

$w \approx 2.90\ ft$

$l \approx 5.80\ ft$

$h = 3.86\ ft.$