

Calculus I SI Worksheet

Section 3.5: Implicit Differentiation

1. $\frac{d}{dx} (2xy^2 + 3y - 5x = 17)$

$2x$	y^2
2	$2y \frac{dy}{dx}$

$$2y^2 + 4xy \frac{dy}{dx} + 3 \frac{dy}{dx} - 5 = 0$$

$$\frac{dy}{dx} (4xy + 3) = 5 - 2y^2$$

$\frac{dy}{dx} = \frac{5 - 2y^2}{4xy + 3}$
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2. $\frac{d}{dx} (\sin(x) + \cos(y) = 3xy)$

$3x$	y
3	$\frac{dy}{dx}$

$$\cos(x) - \sin(y) \frac{dy}{dx} = 3y + 3x \frac{dy}{dx}$$

$$\frac{dy}{dx} (3x + \sin(y)) = \cos(x) + 3y$$

$\frac{dy}{dx} = \frac{\cos(x) + 3y}{3x + \sin(y)}$

3. Find the second derivative of $y^3x + x^3y = 12$

$$3y^2x \frac{dy}{dx} + y^3 + 3x^2y + x^3 \frac{dy}{dx} = 0$$

y^3	x
$3y^2 \frac{dy}{dx}$	1

x^3	y
$3x^2$	$\frac{dy}{dx}$

$$\frac{dy}{dx} (3y^2x + x^3) = -y^3 - 3x^2y$$

$$\frac{dy}{dx} = \frac{-y^3 - 3x^2y}{3y^2x + x^3}$$

$y^3 - 3x^2y$	$3y^2x + x^3$
$3y^2 \frac{dy}{dx} - 6xy - 3x^2 \frac{dy}{dx}$	$6xy \frac{dy}{dx} + 3y^2 + 2x^2$

$$\frac{d^2y}{dx^2} = \frac{(3y^2x + x^3) \left(3y^2 \frac{dy}{dx} - 6xy - 3x^2 \frac{dy}{dx} \right) - (y^3 - 3x^2y) (6xy \frac{dy}{dx} + 3y^2 + 2x^2)}{(3y^2x + x^3)^2}$$

$3x^2$	y
$6xy$	$\frac{dy}{dx}$

$3y^2$	x
$6y \frac{dy}{dx}$	1

$\frac{d^2y}{dx^2} = \frac{(3y^2x + x^3) \left[(3y^2 - 3x^2) \left(\frac{-y^3 - 3x^2y}{3y^2x + x^3} \right) - 6xy \right] - (y^3 - 3x^2y) \left[6xy \left(\frac{-y^3 - 3x^2y}{3y^2x + x^3} \right) + 3y^2 + 2x^2 \right]}{(3y^2x + x^3)^2}$
