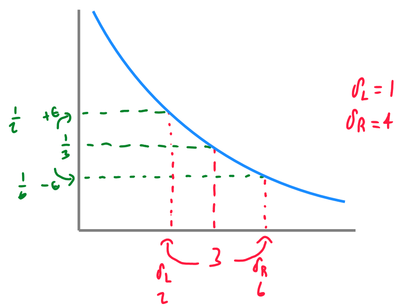
Lesson 2.4: The Precise Definition of a Limit

Find a number such that:

If $0 < | x - 3 | < \delta$ then $| \frac{1}{x} - \frac{1}{3} | < \frac{1}{6}$

L	ε	a	δ	f(x)
1/3	1/6	3	1	1/x



$\delta_L = 1$
$\delta_R = 4$

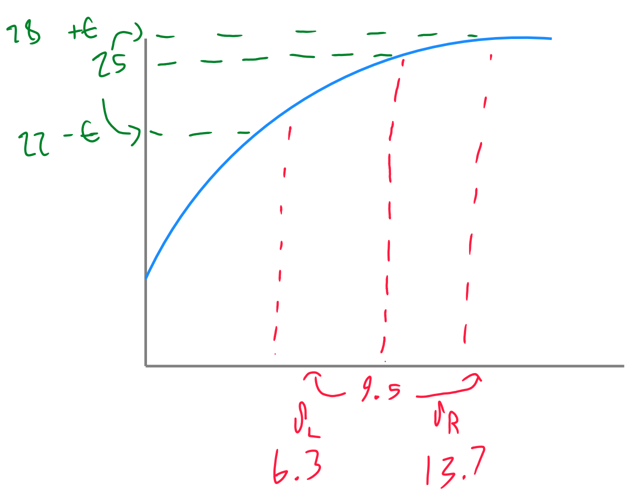
$\delta = \min[\delta_L, \delta_R]$
$\delta = \delta_L = 1$

The growth of the population of eagles can be modeled using the function:

$$f(t) = 5\ln([t + 2.72]^2)$$

Where t is the time in years. A conservationist wants to know when the population will reach 25 in order to begin transporting eagles to prevent overpopulation. If it is allowable for the population to be within 3 of this number, how far from the target date can the conservationist begin relocating eagles?

L	ε	a	δ	f(x)
25	3	9.5	3.2	$5\ln([t+2.72]^2)$



$25 = 5\ln(t+2.72)^2$
$5 = \ln(t+2.72)^2$
$e^5 = (t+2.72)^2$
$\sqrt{e^5} - 2.72 = t = 9.5$

$22 = 5\ln(t+2.72)^2$
$t_L = 6.3$
$\delta_L = 3.2$

$28 = 5\ln(t+2.72)^2$
$t_R = 13.7$
$\delta_R = 4.2$

$\boxed{\delta = 3.2}$ years

A manufacturing company has an order to produce rectangular containers with the following parameters:

- The capacity of the crates should be 1000L ($1m^3$)
- The height and width should be equal
- The length should be twice the width

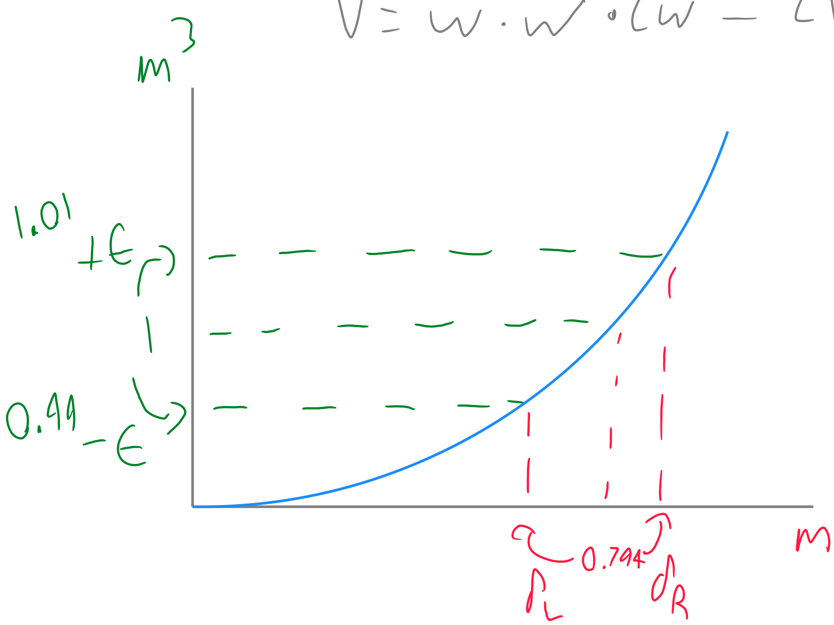
Suppose the client has agreed to a 1% tolerance in the volume of the crate. Calculate the δ for the width in order to remain compliant with the client's demands.

L	ε	a	δ	f(x)
1	0.01	0.794	0.002	$2w^3$

$V = h \cdot w \cdot L$

$h = w \qquad L = 2w$

$V = w \cdot w \cdot 2w = 2w^3$



$1m^3 = 2w^3$

$w = 0.794$

$0.99 = 2w^3 \qquad 1.01 = 2w^3$

$w_L = 0.791 \qquad w_R = 0.796$

$\delta_L = 0.003 \qquad \delta_R = 0.002$

$\boxed{\delta = 0.092m}$